The 8th International Scientific Conference
eLearning and software for Education
Bucharest, April 26-27, 2012
10.5682/2066-026X-12-084

FROM LINEAR TO NONLINEAR PHENOMENA THROUGH COMPUTER SIMULATION

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Abstract*: In order to better grasp the natural phenomena, we must recognize the roll of nonlinear effects as a major improvement for describing reality. Accordingly, we have as a purpose the development of a gradual introduction of the so-called "nonlinear dynamic systems" as a turning point for better understanding the nature. The true value of the computer approach is revealed for solving, modelling and/or simulating nonlinear phenomena. In order to do that, we are preparing a series of lectures and applications for different phenomena that are simple enough to understand but difficult to compute, in a gradual way. We analyze a series of phenomena, and develop algorithms for computing or simulating them. The project also includes instructions for teaching as well as examples and proposes a thematic for further developments. It is not intended to fully develop all the phenomena, but to open a road that should be followed for all types of problems of main interest. Among these we can mention the following: oscillators of different kinds, linear, coupled, collection of oscillators, non-linear oscillators, damping and non-autonomous oscillators, exemplified by mechanic, electric or optical systems. We will also develop simulations for Verhulst dynamics, logistic developments, pray-predator systems and not only. The whole project involve a series of lectures developed and based mainly on the Romanian curricula and on the textbooks that exist in the bookshops [9], [10], [21].*

Keywords: *dynamical systems; nonlinearity; teaching methodology in physics*

I. PHYSICS AS A MAIN ROOT FOR INTERDISCIPLINARY UNDERSTANDING THE NATURE

The remarkable development of science and technology is a direct results of the deeper insight of the nature and his "mechanisms" [8]. We are now able to understand on a much larger and unified background, the natural phenomena. Still our educational methods, curricula, and view are still over simplified. The difficulties arise from many directions: the complexity of the real phenomena, the large amount of new knowledge and ideas, the increase of the accuracy of the computational approach that can bring and reveal much "finer" aspects, of the reality, the presence of the highly sophisticated devices in our everyday life, the increasing "simplicity" of using "intelligent" devices, produce – for a unaware person - a wrong idea of "simplicity" of the science. From here, the students often conclude that learning could be easier achieved than in the past. Also that is accompanied with a lack of capacity of spending time for not to simple problems. As a results their knowledge diminish and their ability to cope with the real difficult problems are very low, except a small number of dedicated/gifted students. Also developing an interdisciplinary education, specific for the future needs, we have to be able to cope with the diversity of the nature, and that is not a simple task. Examples and basis for transdisciplinar approach could be develop using computer enhance education [12], [13].

We are aware of this decrease in knowledge and capability exact when the human knowledge increase dramatically. These bring a superficial view of the complexity of nature and the technology. The modern themes bring together not only physics knowledge but biology, chemistry, economy, and technology. An approach of the real problems is difficult because not everyone is an expert in physics and biology and fluid mechanics… so science as well as mathematics will need to be explained from scratch. But that should be fun, and it cam be instructive to see the connections among different fields.

II. SYSTEMIC APPROACH OF PHENOMENA: A DYNAMIC VIEW OF THE WORLD

In the traditional teaching of physics we are dealing with the simplest objects, usually material points, or very simple systems [2]. But this give today the “archaic” filling of physics teaching, in the realm of modern science. From our experience, students have difficulties of grasping with “many bodies” or systems. Indeed there are difficulties, and our idea and work is dedicated to decrease of these difficulties. Also the intimate interconnections of the different sciences that today is the most powerful way of understanding the nature, is to be achieved through education.

So, we started by defining the most important concepts needed to understand the new science, as a set of notions that becomes much more important today than yesterday. Sometimes this could be seen as an unnecessary and time vesting stage, but we consider this as a key part of our approach: from very simple ideas to complex one.

First we have to understand that all the phenomena we study belongs to a general subject known as *dynamics* [2], [3]. This is a subject that deals with change, with systems that evolve in time. Whether the system in question settles down to equilibrium, keeps repeating in cycles, or does something more complicated, it is dynamics that that we use to analyse the behaviour. Classical mechanics, chemical kinetics, population biology, astrophysics, differential equations, and so on, could be placed in a common framework and viewed from the perspective of dynamics.

All the systems could be divided in two classis: linear and nonlinear. Also a system has a number of variables needed to characterize the state of the system. So, for introducing the idea of linear or nonlinear system we have to define the notions we are dealing of.

Today dynamics is an interdisciplinary subject, but it was originally a branch of physics that begins around 1660 when Newton invented differential equation and discovered the low of motion, the universal gravitation, and was able to explain Kepler’s lows of planetary motion. That was specifically a two-body problem. After decades of effort from mathematicians and physicist to extend and solve the three-body problem (for example the Sun, Moon and Earth), it turns out to be much more difficult to solve. Today we know that the three-body problem essentially *impossible* to solve in the sense to obtain explicit formulas for the motion of three body system. These come with the work of Poincaré from around the 1890. It was him that changed the point of view, from quantitative to qualitative. Instead to ask for the exact position of the planets at all time, he asked “Is the solar system stable forever, or will some planets eventually fly off to infinity?”

The invention of electronic computers in the 1950s changed and boosts dynamics. Simulations and computing becomes a new way of experimenting physics. The discovery by Lorentz in 1963 of chaotic motion (for a simplified model of convection rolls in the atmosphere) was accompanied by the observation that the solutions of the equations never settled down to equilibrium or to a periodic state, instead they continue to oscillate in an irregular, aperiodic fashion. The implication was that the system was *inherently* unpredictable [4], [5]. The only way left for science of nature was to use computers, to simulate and model complex system and to analyse the precision and errors involved – for long terms – in these predictions. This has to be introduced to students of our days.

But using computers and entering in the realm of computing, we can improve our teaching and learning capability because we slowly approach the mechanism of cognition, memory and intelligence, and that could give us an increase efficiency in education [11].

III. LINEAR AND NONLINEAR SYSTEMS

Since the Newton description of the dynamical systems by differential equations (DE), is the key of understanding nature: first the DE must be mastered and after it has to be solved [1], [3]. A nonlinear system is described by a nonlinear differential equations. Lets have some examples. A linear system is described by a linear DE, and a nonlinear system by a nonlinear differential equations like:

$$\frac{d^2x}{dt^2} + x = 0 \quad \text{linear DE;} \qquad \frac{d^2x}{dt^2} + x^3 = 0 \quad \text{nonlinear DE} \tag{1}$$

Other examples (that we developed with appropriate teaching methodology) are for elementary oscillators:

$$m\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + kx = a\cos\Omega t \qquad \text{forced, damped linear oscillator} \tag{2}$$

$$m\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + kx^3 = a\cos\omega t \qquad \text{forced, damped nonlinear oscillator} \tag{3}$$

Two problems arise here. First is the difficulty of understanding the DE at the undergraduate level (otherwise known and classical situation for physics), and second the method of solving the DE (known in mathematics and for physics at the higher level). First problem was solved by simple step-by-step building the form of the DE using physical arguments (experimental and theoretical one). The second problem was solved by devising algorithm for solving DE, using a step-by-step approach. The method of solving DE was the development of the idea of Newton method of fluxions (1687 - the concept of derivative) and using the simple straightforward Euler's method (1749) of a crude approximation (figure 1)

$$\frac{dx}{dt} = \frac{x_{new} - x_{old}}{\Delta t} \tag{4}$$

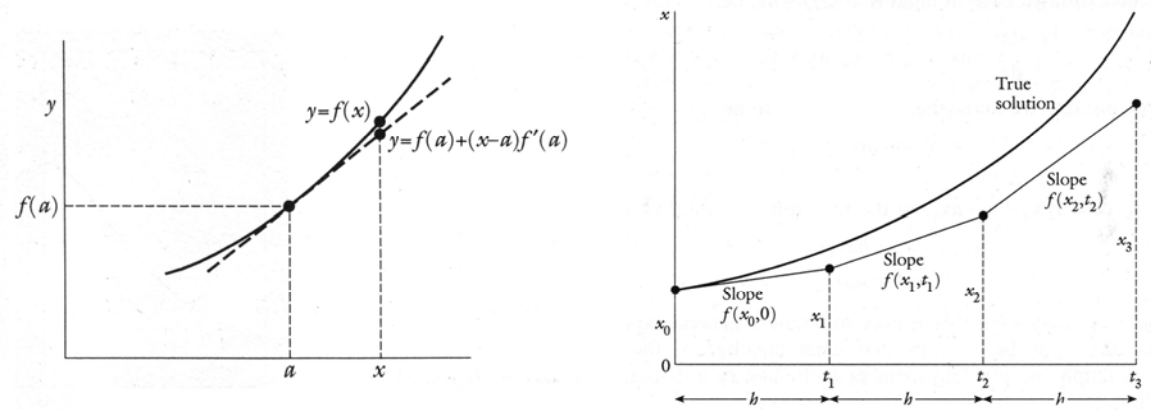

Figure 1. Illustrating steps for Euler's method

For a teacher or for a scientist, these are well-known procedures, but for methodological point of view this is a crucial for teaching. The idea is to device a "dialog" with the students that are clear, simple, and not a boring presentation. So, this can be done first be the method of "doing" by hand. The Euler's method is suitable for direct calculation and is easy to exemplify on the problem of the discharge of a capacitor in a RC circuit. Using Ohm's law, the definition of electric current and the definition of the capacitance we obtain:

$$R\frac{dq}{dt}+\frac{q}{C}=0 \qquad \text{that becomes after rearranging} \qquad \frac{dq}{dt}=-\frac{q}{RC}=\frac{q}{T} \tag{5}$$

Euler techniques convert the DE into a *finite difference equation* substituting the ratio *dq/dt* by finite quantities:

$$\frac{\Delta q}{\Delta t}=\frac{q(t+\Delta t)-q(t)}{\Delta t} \qquad \text{and} \qquad \Delta q=-\frac{\Delta t}{T}q \tag{6}$$

If the initial charge q_0 is known, repeated use of equation (6) permits us to find the charges at a series of times t = Δt, 2Δt, 3Δt, 4Δt, …. To illustrate the procedure, we assume q = 1, T = 10 and Δt = 1. Appling (6) the change in charge after t = Δt is Δq = -1/10 = -0,1. The amount of charge remaining at the end of first time interval will be: $q_1 = q_0 + \Delta q = 0{,}9$. During the next interval the change of the charge will be Δq = -1/10 ×0,9 = -0,09. The amount of charge remaining after the second interval of time is $q_2 = q_1 + \Delta q = 0{,}81$. By repeating this procedure many times we obtain the series of the values for the charge: 1,0; 0,9; 0,81; 0,729; 0,656; 0,59; ……

Comparing these results with the exact formula that describe the discharge, we see differences (the formula gives: 1,0; 0,904; 0,818; 0,740; 0,670; 0,606; ……). As we pointed out that the precision is not important for the understanding and grasping the phenomena. If we want and if we have students that are willing to obtain better results, we can explain and introduce *Improved Euler's Method*, or even *Runge-Kutta Method*, that are suitable for exact computations and also could be found implemented in professional math codes like *Mathematica* or *MathCAD.*
Alternative we can use for students tabular computing (using *Excel*) , or other codes [16-18].

The idea is that this could be done as a simple exercise in the class using hand-held computers. It is simple, straightforward, easy to "see" the phenomenon of the discharge of the capacitor. This procedure could be use now for a lot of other phenomena form physics, governed (we now!) by exponential law of variation like: radioactive decay, light attenuation when traverse the atmosphere (or other transparent bodies), decrease of the atmospheric pressure with altitude (barometric formula), sedimentation, but also very "domestic phenomena" like population growth (Verhulst dynamics), interest, capital budgeting, linear prediction, and so on.

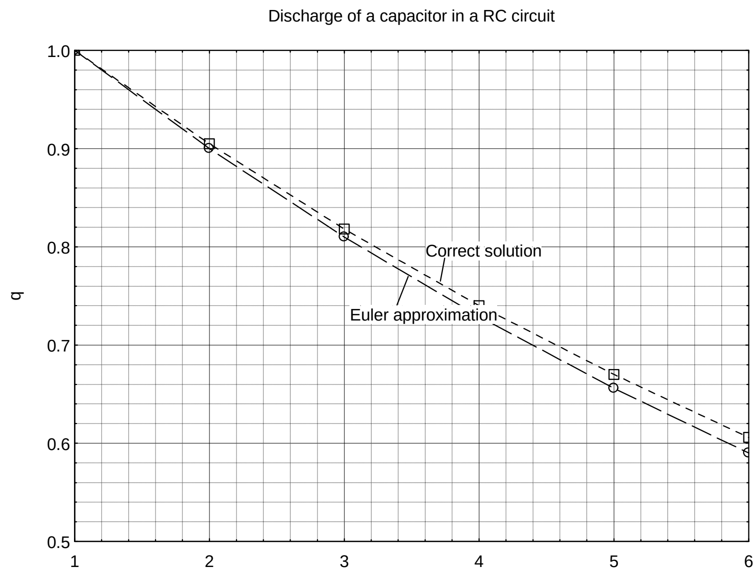

Figure 2. Comparing approximation with the exact formula

The second advantage of using simple method of solving DE is that we can approach in a simple way a lot of problems described in nonlinear dynamics like chaos, strange attractors, intermittence, sensitivity to the initial condition, phase transition, far from equilibrium dynamics, fluid dynamics and turbulence, and a lot of other phenomena that are difficult to understand and sometimes to believe. But all these themes gives a new and useful scenery of the modern science.

But not only ordinary differential equations are related to nonlinearity; also partial differential equation (PDE) that describe other important phenomena la propagation of waves (acoustic, mechanic, electric, electromagnetic – for example antennas), solar wind, earthquake wave propagation, tsunami, building up phenomena like epidemic phenomena, forest fires, or the stability of the buildings, bridges (the Takoma bridge), and so on, are closely related to nonlinearity and imperceptibility that describe majority of natural phenomena (weather, earthquake, volcanic eruption, solar wind and space weather, dynamic of market and so on.

So, we argued here that we have to have the courage to use the simplest (yet not precise – that does not matter here) method as an introductory, cognitive step of learning. In other words we try to use a "natural" way of empirical (heuristically and sometime historical) learning and understanding

IV. COMPUTER APPROACH OF TEACHING PHYSICS

What's wrong with programming Today? Programming should be simple in order to do a lot with less work and more enjoyment! Usually the programming software can cost a lot, so it is advisable to have one which is not too expensive. For maintaining interests in physics or science it is advisable that students can write their first program in minutes. If students spend more time doing things that enjoy, they will be strait to the object: interpretation and not banging their heads in frustration. We developed simple computer codes using computer languages that students from high school learn or could learn in a simple way. These practice will be also a application of their knowledge in programming. We choose to start with QBASIC. It is simple enough to understand and not difficult to compute. So we choose Liberty BASIC that gives you also a power toolkit for Windows programming, www.libertybasic.com. It works with Windows 95 or 98, so we can use old (inexpensive) computers, because the idea is not to have performance in computing but to have speed and fun in understanding physics examining the results and spending time on interpreting the results. Problems on using QBASIC could be found in http://www.computing.net/answers/dos/quickbasic-45/3981.html. We used and adapted a lot of other codes written by different authors or colleagues, in order to simplify the understanding the results of the computing [5], [16 - 20]. These codes are used as a library for teaching and learning, like books from a library or practical activity.

Another interesting feature, that is possible to experiments having skills and easiness to program, is to enter the world of virtual "universe" in which you can "construct" strange, unusual, worlds, using different kind of formulas. For example you can study a word of aerodynamic friction in which the drag depends on speed at an arbitrary exponents for example: $v^{5,63}$ or other, or a motion in a gravitational field that is not a Newtonian type $1/r^2$ but $1/r^3$. The last one could be a SF-world and you can see the consequences on the motion. You will understand how strange the motion is, how unreal the motion appear, from that we can conclude that the Newton low for gravitation is the only one to fit the reality. This conclusions are very powerful results, for learning and understanding our world. Experiencing other laws, indirectly we prove the theories that they learn. It is a serious game activity that enhance understanding, and also creativity, on a more scientific basis.

We don't exclude to use higher level programming or using Virtual Instrumentation [6] or LabVIEW [7], but that we propose to be used after the student understand the "black box" of the computing "laboratory". If they reach such a level, sensors, and experimental devices can be use for more serious investigations (for example for weather monitoring, biology experiments, earthquake).

V. CONCLUSIONS

We present here the aim and the idea that more complicated system could be much easier explained than expected, in order to improve the understanding of the common natural phenomena, in spite of their complexity. The main idea is to start from the very, very simple systems and concepts, and on the other hand to give students a way of doing computer experiments by their own ideas and possibilities. We expect that our approach to widen the knowledge of students, without creating more difficult tasks, and also to attract them to study natural sciences. We are continuous improving and completing the codes, materials, phenomena and dynamical system that could be used for teaching.

Acknowledgements

Thanks to Prof. Dr. E.S. Barna from Physics Faculty, University of Bucharest, for valuable observations and discussions (and who is guiding my activity in order to finish the PhD thesis).

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